Review Session 1 Problems

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Problems we discussed:

1. Prove the closed-form formula for $\sum_{j=1}^{n} j^2$ (from your textbook).

   Idea: Set S_n to be the sum up to n, then compute $S_{n+1} - S_n$.

2. Show $\sqrt{3}$ is irrational. We also discussed why $\sqrt{p}$ is irrational for any prime p (from your textbook).

   Idea: Suppose $\sqrt{p} = r/s$ for coprime $r, s > 0$, use algebra to show that p would be a common factor of both.

3. Show that for any real numbers $a < b$, there exists an irrational number ζ such that $a < \zeta < b$ (from Wade).

   Idea: Approximate a by a larger rational a', and b by a smaller rational b'. Then $\zeta = a' + \frac{\sqrt{2}}{2}(b' - a')$ is an irrational number satisfying $a < \zeta < b$.

4. Find the supremum and infimum of $\{p/q \in \mathbb{Q} : p^2 < 2q^2, p, q > 0\}$, $\{x \in \mathbb{R} : x = 1 + (-1)^n, n \in \mathbb{N}\}$, $\{x \in \mathbb{R} : x = 1/n - (-1)^n, n \in \mathbb{N}\}$, $\{1 + (-1)^n/n : n \in \mathbb{N}\}$ (from Wade).

   Idea: If you don't immediately see what the answer should be, graph the sequences. Once you have a good guess for the infimum and supremum, you still have to prove it's correct!

Problems we didn't get to discuss:

1. Suppose $E, A, B \subset \mathbb{R}$, and $E = A \cup B$. Prove that if E has a supremum and both A and B are nonempty, then $\sup A$ and $\sup B$ both exist, and $\sup E$ is either $\sup A$ or $\sup B$ (from Wade).

   Idea: If $\sup A > \sup B$, then $\sup A > x$ for any $x \in B$.

Note: Wade = "Introduction to Analysis" by William R. Wade.