Review Session for Dr. Thomas's Exam I

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Problems we discussed:

1. Show that a set of integers that is bounded from below has a minimum.

   Idea: Consider the negative of the set, note that it's bounded from above and therefore has a supremum. Then show this supremum is the maximum.

2. Is $\{\frac{n}{10^m} | n, m \in \mathbb{Z}, n, m \text{ are odd}\}$ dense in $\mathbb{R}$?

   Idea: Take $a, b \in \mathbb{R}$. Find an odd m so small that $1/10^m$ is much smaller than $b - a$.

3. Let $x_n \geq 0$ be a bounded sequence. What is the infimum of $\{x_n/n | n \in \mathbb{N}\}$?

   Idea: Since $x_n \leq M < \infty$ for some M, we have that $x_n/n \leq M/n$ for any n. Since we can make n arbitrarily large, x_n/n can get arbitrarily close to 0, and therefore 0 is the infimum.

4. Show that the Archimedean property by the statement that every interval (a, b) contains a rational number.

   Idea: Let $c > 0$. Take a rational number $q \in (c + 1, c + 2)$, then look at its integer part, $\lfloor q \rfloor$. We then have $\lfloor q \rfloor \geq c + 1 - 1 = c$ (Q: can we replace $\geq$ with $>$?).

5. Infimum/supremum questions from review session 1.

Good luck on your exam!